



Demultiplexing Generalized Information via Quantum Transmission Lines

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arXiv link

DEMULPLEXERS

- Classically, a demultiplexer is a device with a single classical input port with an n -bit string X^n and with m possible output ports, each capable of storing an n -bit classical message. The sender has an access to an additional $\lceil \log_2 m \rceil$ -bit strings, namely the selectors, would be able to transfer the input classical message X^n in any of the desired output ports among $\{1, 2, \dots, m\}$.

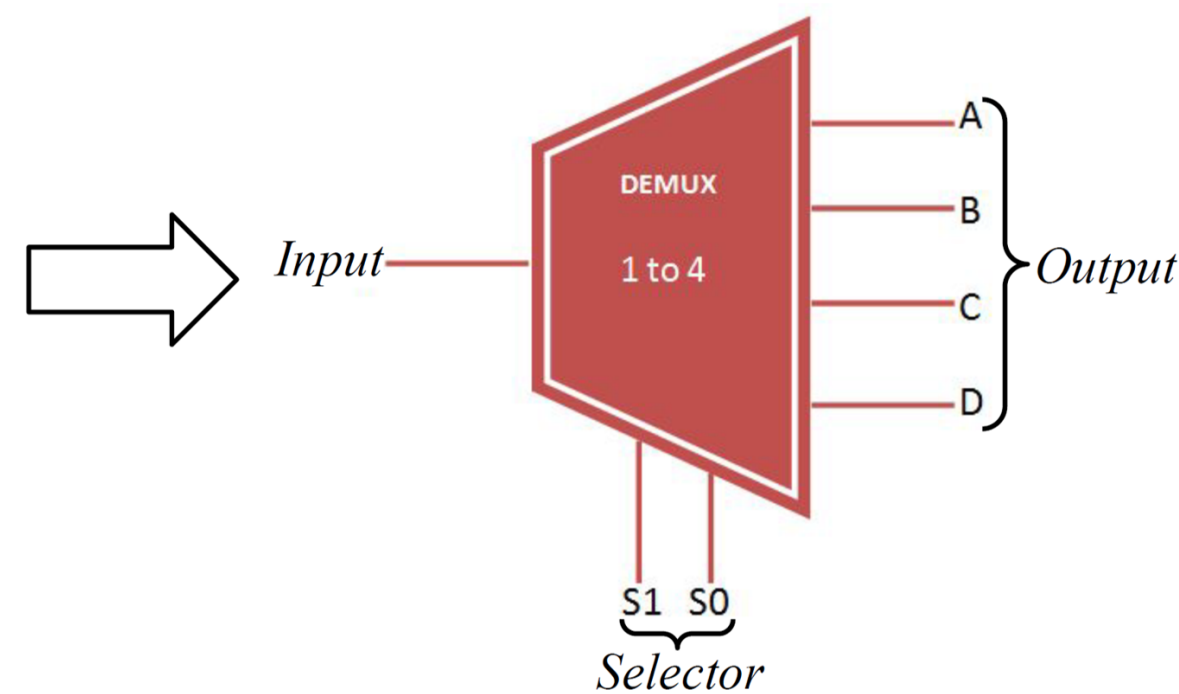


Figure 1: Representation of a classical Demultiplexer.

QUANTUM DEMULPLEXERS

- We propose a quantum demultiplexer (Q-DEMUX), which is capable of separating the classical or the quantum information stored in the input quantum system.
- This implies a Q-DEMUX D can be operationally seen as a device with a single input quantum port (A) along with two kinds of output ports: the first one (Q) stores the quantum information, while the second one (C) is a classical register only. The sender, by tuning a binary selector $s \in \{0, 1\}$ is able to transfer the information in the designated port depending upon its nature.

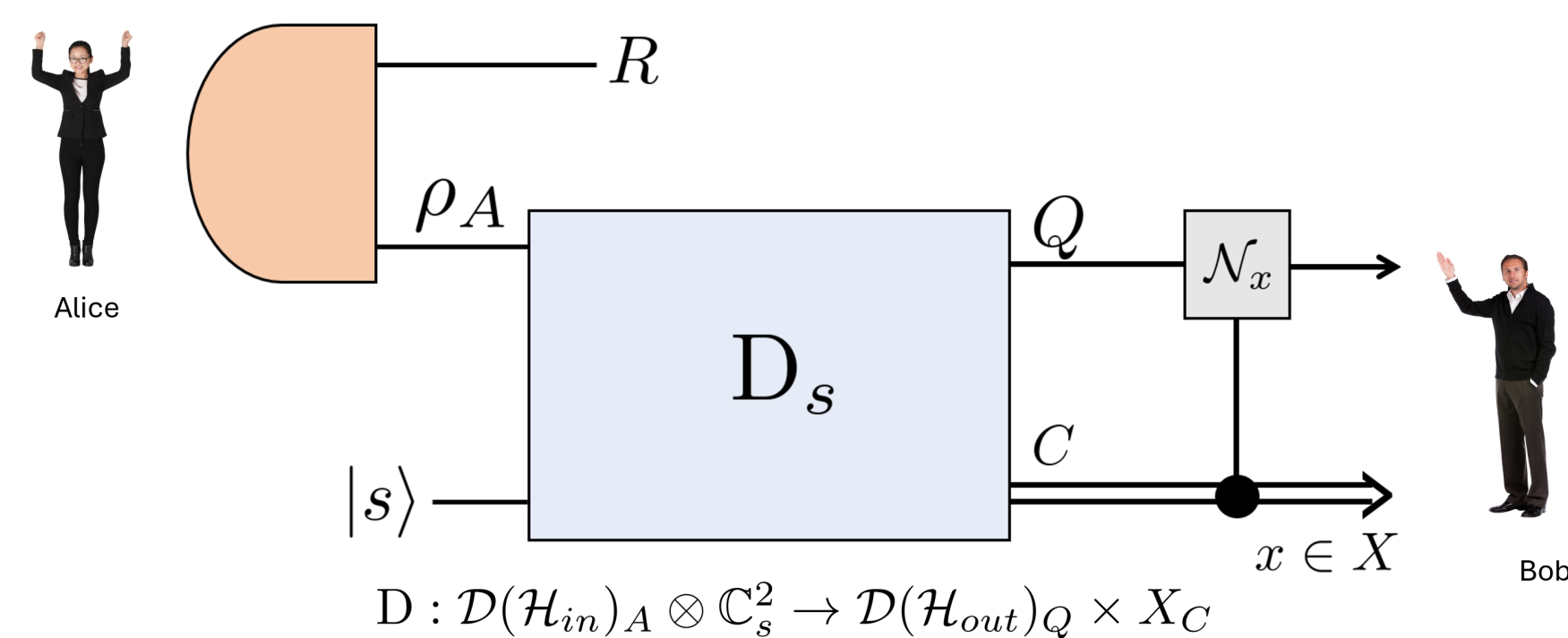


Figure 2: Schematic diagram of a Q-DEMUX. Here, we also append the role of the selector qubit.

CLASSICAL CAPACITY OF Q-DEMUX

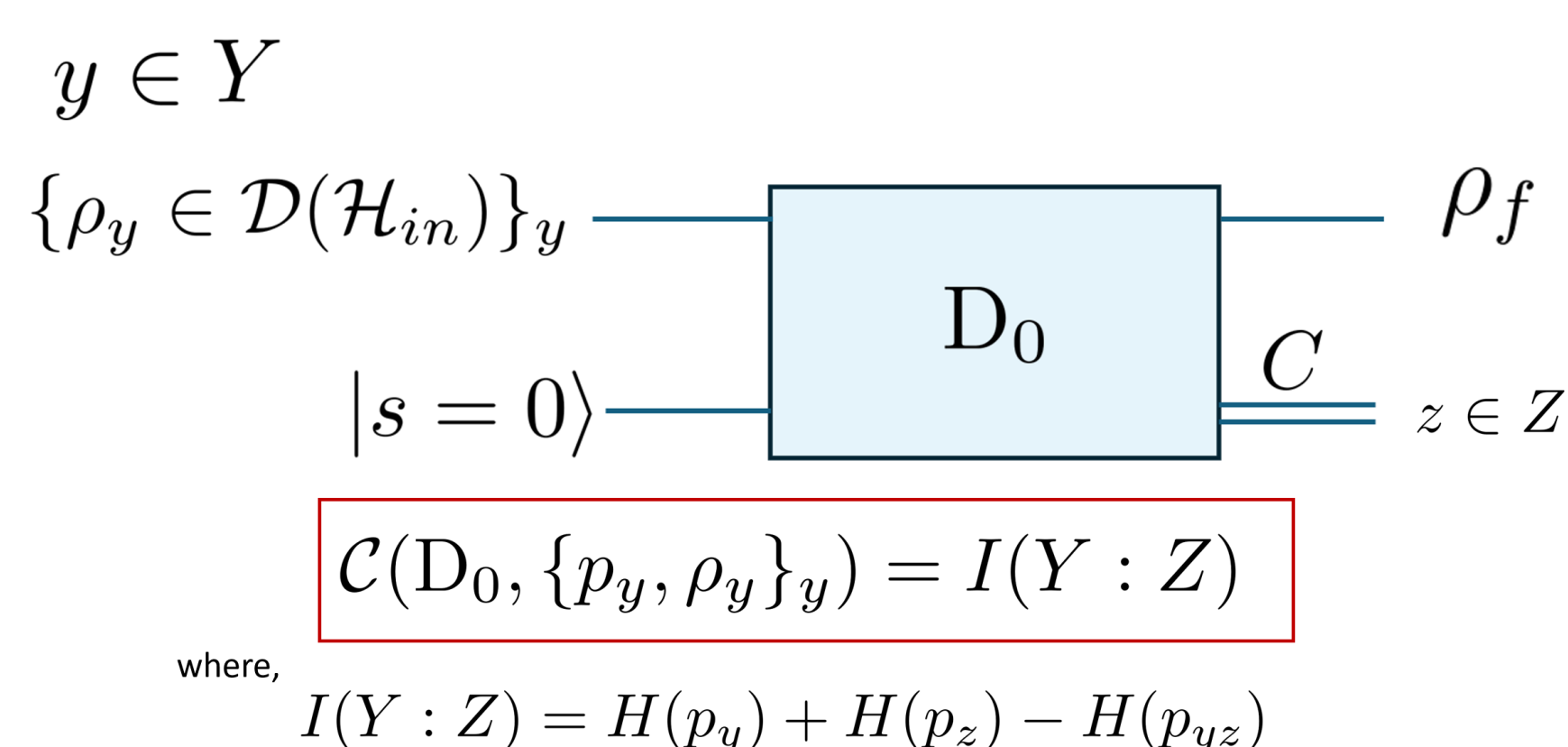


Figure 3: Here, the suffix "0" for D denotes the choice of the selector $s = 0$ in context of classical information processing.

QUANTUM CAPACITY OF Q-DEMUX

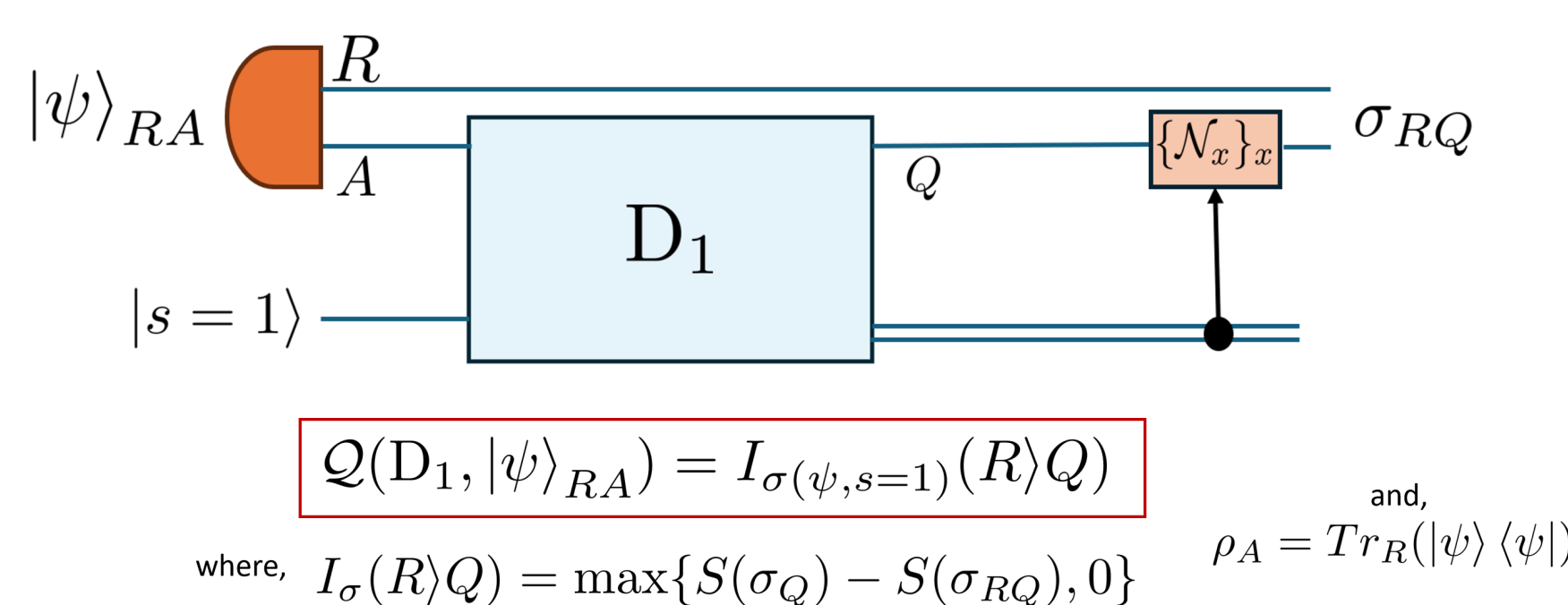


Figure 4: Here, the suffix choice is "1", which denotes the choice of the selector $s = 1$, in the context of quantum information processing. Also, the effective quantum demultiplexing strength of D_1 with the specific $\rho_A \in \mathcal{D}(\mathcal{H}_{in})$, optimizing over all possible correcting operations which we have omitted here for the sake of simplicity.

TOTAL DEMULPLEXING STRENGTH

Within this framework we can now, define the total demultiplexing strength of a Q-DEMUX D_s as

$$\mathcal{T}_s(D_s) = \max_{\rho_A \in \mathcal{D}(\mathcal{H}_{in})} [\mathcal{C}(D_0, \rho_{RA}) + \mathcal{Q}(D_1, |\psi\rangle_{RA})], \text{ with } \rho_A = \text{Tr}_R(\rho_{RA}) = \text{Tr}_R(|\psi\rangle\langle\psi|_{RA})$$

While to make it more operationally feasible one may come up with various possible constraints on the architecture of the Q-DEMUX, here we consider a fairly simple one: *The quantum channel induced from the Q-DEMUX must be invariant under the choice of the selector.*

This condition mathematically reads

$$\text{Tr}_C[D_s(\rho_A)] = \sum_x p(x, \rho, s) \sigma(x, \rho, s)_Q = \sigma(\rho)_Q,$$

for every possible input quantum state $\rho_A \in \mathcal{D}(\mathcal{H}_{in})$ and for every choice of the selector $s \in \{0, 1\}$. We can identify the above operation as a CPTP-map $\Lambda_{ind} : \mathcal{D}(\mathcal{H}_{in}) \rightarrow \mathcal{D}(\mathcal{H}_{out})$, namely the induced quantum channel.

QUANTUM INSTRUMENTS

We formulate the Q-DEMUX mathematically in terms of quantum instruments, which are $\mathcal{D}(\mathcal{H}_{in}) \rightarrow \mathcal{D}(\mathcal{H}_{out})$ with outcomes in a (finite) set X is a collection of completely positive trace non-increasing (CPTNI) maps $\mathcal{I}_x : \mathcal{D}(\mathcal{H}_{in}) \rightarrow \mathcal{D}(\mathcal{H}_{out})$ such that $\mathcal{I}_{ind} := \sum_x \mathcal{I}_x$ is trace preserving. Alternatively, an instrument can be viewed as a channel with both quantum and classical outcomes, that is

$$\mathcal{I}(\rho) = \sum_x \mathcal{I}_x(\rho) \otimes |x\rangle\langle x|.$$

Lemma 1. Any Q-DEMUX D_s can be configured in terms of two instrument realizations of the induced quantum channel Λ_{ind} .

MAXIMIZING THE DEMULPLEXING STRENGTH

Lemma 2. For any Q-DEMUX, $D_s : \mathcal{D}(\mathcal{H}_{in}) \rightarrow \mathcal{D}(\mathcal{H}_{out}) \times X$, with an access to the selector, the total demultiplexing strength satisfies $\mathcal{T}_s(D_s) \leq 2 \log_2 d_{in}$.

We characterize the class of quantum channels that have a Q-DEMUX realization that achieves the bound.

Proposition 1. A quantum channel Λ_{ind} has a Q-DEMUX realization D_1 with $\mathcal{Q}(D_1) = \log_2 d_{in}$ if and only if Λ is a convex combination of isometries (RI) $\{\mathcal{V}_k : \mathcal{H}_{in} \rightarrow \mathcal{H}_{out}\}_k$. This also implies that for $\mathcal{H}_{in} \simeq \mathcal{H}_{out}$, Λ is a random-unitary channel.

Intuition: For channels $\Lambda_{ind}(\rho) = \sum_k p_k \mathcal{V}_k \rho \mathcal{V}_k^\dagger$ you can measure the classical index j and correct with the corresponding correction operation $\mathcal{N}_j$

Proposition 2. A quantum channel Λ_{ind} has a Q-DEMUX realization D_0 with $\mathcal{C}(D_0) = \log_2 d_{in}$ if and only if Λ is a classical-quantum (C-Q) channel.

Intuition: For channels like: $\Lambda_{ind}(\rho) = \sum_k \langle k | \rho | k \rangle \sigma_k$ Encode the classical messages in O.N. Basis and store the projective measurement result in the classical register.

Theorem 1. A quantum channel Λ_{ind} admits a Q-DEMUX realization D_s with $\mathcal{T}_s(D_s) = 2 \log_2 d_{in}$, if and only if Λ_{ind} is both classical-quantum and a random isometry (CQ-RI).

Q-DEMUXS WITHOUT ACCESS TO THE SELECTOR

Selector-less demultiplexing strength :

$$\mathcal{T}(D) = \max_{\rho_A \in \mathcal{D}(\mathcal{H}_{in})} [\mathcal{C}(D, \rho_{RA}) + \mathcal{Q}(D, |\psi\rangle_{RA})]$$

Theorem 2. For any Q-DEMUX $D : \mathcal{D}(\mathcal{H}_{in}) \rightarrow \mathcal{D}(\mathcal{H}_{out}) \times X$, the selector-less total demultiplexing strength $\mathcal{T}(D) \leq \log_2 d_{in}$.

OPERATIONAL INTERPRETATION OF INCOMPATIBILITY OF QUANTUM INSTRUMENTS

$\mathcal{I}(\Omega, \mathcal{H}_{in}, \mathcal{H}_{out})$ and $\mathcal{J}(\Omega', \mathcal{H}_{in}, \mathcal{H}_{out})$ are traditionally compatible, if there exists a joint instrument $\mathcal{G}(\Omega \times \Omega', \mathcal{H}_{in}, \mathcal{H}_{out})$, such that

$$\sum_{y \in \Omega'} \mathcal{G}_{x,y} = \mathcal{I}_x \quad \forall x \in \Omega, \text{ and } \sum_{x \in \Omega} \mathcal{G}_{x,y} = \mathcal{J}_y \quad \forall y \in \Omega'.$$

Theorem 3. For any Q-DEMUX $D : \mathcal{D}(\mathcal{H}_{in}) \rightarrow \mathcal{D}(\mathcal{H}_{out}) \times X$, if the selector can access two instrument realizations of Λ_{ind} to achieve $\log_2 d_{in} < \mathcal{T}_s(D) \leq 2 \log_2 d_{in}$, then they must be traditionally incompatible.

Graphical representation of $\mathcal{T}_s(D)$ vs d

